1996–97 # American Computer Science League **Contest #3**

Junior Division — SHORT PROBLEMS

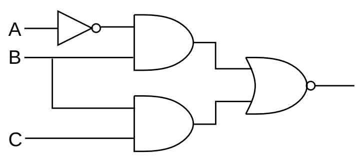
1. What Does This Program Do? — Subscripted Variables

Variable `x` is a 5x5 array, initially set to all zeros. How many elements of the array will be non-zero after the following program runs?

```
for r = 2 to 4
  for c = 2 to 4
    x(r+1,c+1)=1
    x(r-1,c-1)=1
  next c
next r
```

2. Elementary Digital Electronics

List all of following expressions that are exact translations of the circuit at the right.



(a) $\overline{\overline{A}B + BC}$

(b) $(A + \overline{B})(\overline{B} + \overline{C})$

(c) $\overline{(\overline{A} + B)(B + C)}$

(d) $\overline{CB + \overline{A}B}$

(e) $A\overline{B} + \overline{BC}$

3. Elementary Digital Electronics

Draw a digital circuit that is equivalent to the following expression:

$$\overline{(A + B)C}$$

4. Recursive Functions

Find $f(10)$, given the following:

$$f(x) = \begin{cases} f(x-7)+3 & \text{if } x > 6 \\ f(x-3)+1 & \text{if } 2 < x \le 6 \\ x-2 & \text{otherwise} \end{cases}$$

5. Recursive Functions

Find $f(7)$, given that $f(1) = 2$, $f(2) = 4$, and

$$f(n) = \frac{f(n-1) \cdot f(n-2)}{2}$$

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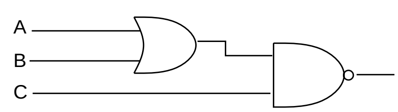
Junior Division — SOLUTIONS

1. The following table shows which elements of `x` are set to 1 each time through the inner loop: r | c | 2 | 2 | (3,3) (1,1) 2 | 3 | (3,4) (1,2) 2 | 4 | (3,5) (1,3) 3 | 2 | (4,3) (2,1) 3 | 3 | (4,4) (2,2) 3 | 4 | (4,5) (2,3) 4 | 2 | (5,3) (3,1) 4 | 3 | (5,4) (3,2) 4 | 4 | (5,5) (3,3) Note that `x(3,3)` appears twice.	17
2. The circuit translates to $(\overline{A}\text{ AND }B)\text{ NOR }(B\text{ AND }C)$ which can be written as $\overline{\overline{A}B+BC}$ (choice (**a**)). Choice (**d**) is the same, just with some operators commuted. Choice (**b**) is derived from choice (**a**) by applying DeMorgan's Law; however, this is not a translation of the circuit. And choices (**c**) and (**e**) are like (**a**) and (**d**), but with the AND or OR operators swapped.	(**a**) *and* (**d**)
3. What's important about the drawing of the circuit is that there is one OR gate and one NAND gate. The inputs to the OR gate are A and B (in either order), and the inputs to the NAND gate are C and the output of the OR gate.	
4. The evaluation is as follows: $f(10) = f(3) + 3$ $f(3) = f(0) + 1$ $f(0) = -2$ Now work backwards: $f(3) = -2 + 1 = -1$, and $f(10) = -1 + 3 = 2$.	2
5. Let's evaluate $f(3), f(4), \ldots, f(7)$: $f(3) = \frac{f(2)\cdot f(1)}{2} = \frac{4\cdot 2}{2} = 4$ $f(4) = \frac{f(3)\cdot f(2)}{2} = \frac{4\cdot 4}{2} = 8$ $f(5) = \frac{f(4)\cdot f(3)}{2} = \frac{8\cdot 4}{2} = 16$ $f(6) = \frac{f(5)\cdot f(4)}{2} = \frac{16\cdot 8}{2} = 64$ $f(7) = \frac{f(6)\cdot f(5)}{2} = \frac{64\cdot 16}{2} = 512$	512