

**1. What Does This Program Do? (BASIC)**

If the following program prints 15, what was the input?

```
10 input N
20 if N > 5 then N = N+8
30 if N > 12 then N = N-6
40 if N >= 8 then N = N+1
50 print N
60 end
```

**2. Computer Number Systems**

How many numbers between 300 and 500, inclusive, have exactly eight 1's in their binary representation?

**3. Computer Number Systems**

What is the base 16 value of  $FEDCBA_{16} - ABCDEF_{16}$  ?

**4. Recursive Functions**

Find  $f(15)$ , given the following:

$$f(x) = \begin{cases} 2 * f(x - 3) - 4 & \text{if } x \text{ is composite} \\ x^2 + x & \text{if } x \text{ is prime} \end{cases}$$

**5. Recursive Functions**

Find  $f(10, 5)$ , given the following:

$$f(x, y) = \begin{cases} f(x - 2, y + 1) + 3 & \text{if } x > y \\ f(x + 1, y - 3) + 2 & \text{if } x < y \\ 5 & \text{otherwise} \end{cases}$$

1996–97 **American Computer Science League** Contest #1  
**Senior Division — SOLUTIONS**

<p>1. Work backwards, one line at a time: Line 40 must be called with <math>N=14</math>.  Line 30 must be called with <math>N=20</math>. Line 20 must be called with <math>N=12</math>.</p>	12
<p>2. Convert 300 and 500 to binary:</p> $300 = 100101100$ $500 = 111110100$ <p>There are 9 bits in both of these numbers, so all the numbers between 300 and 500 also have 9 bits. The 9-bit long binary numbers in this range with eight 1's are as follows:</p> $383 = 101111111$ $447 = 110111111$ $479 = 111011111$ $495 = 111101111$	4
<p>3. Work from the right to the left, borrowing as needed:</p> $1A - F = B$ $1(B - \textit{borrow}) - E = C$ $1(C - \textit{borrow}) - D = E$ $1(D - \textit{borrow}) - C = 0$ $E - B = 3$ $F - A = 5$	530ECB <sub>16</sub>
<p>4. The evaluation is as follows:</p> $f(15) = 2f(12) - 4$ $f(12) = 2f(9) - 4$ $f(9) = 2f(6) - 4$ $f(6) = 2f(3) - 4$ $f(3) = 3^2 + 3 = 12$ <p>Working backwards, we have <math>f(6) = 20</math>, <math>f(9) = 36</math>,  <math>f(12) = 68</math>, and <math>f(15) = 132</math>.</p>	132
<p>5. The evaluation is as follows:</p> $f(10, 5) = f(8, 6) + 3$ $f(8, 6) = f(6, 7) + 3$ $f(6, 7) = f(7, 4) + 2$ $f(7, 4) = f(5, 5) + 3$ $f(5, 5) = 5$ <p>Working backwards, we have <math>f(7, 4) = 8</math>, <math>f(6, 7) = 10</math>,  <math>f(8, 6) = 13</math>, and <math>f(10, 5) = 16</math>.</p>	16