

1. What Does This Program Do? (BASIC)

Variable x is a 5x5 array, initially set to all zeros. How many elements of the array will contain a non-zero after the following program is run?

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for r = 2 to 4
  for c = 2 to 4
    if x(r-1,c-1)=0 then x(r-1,c-1)=1 else x(r-1,c-1)=0
    if x(r, c-1)=0 then x(r, c-1)=1 else x(r, c-1)=0
    if x(r+1,c+1)=0 then x(r+1,c+1)=1 else x(r+1,c+1)=0
    if x(r, c+1)=0 then x(r, c+1)=1 else x(r, c+1)=0
  next c
next r
    
```

2. Boolean Algebra

Simplify the following expression as much as possible. The final answer must use as few parentheses as possible and must not have any negation.

$$(A\bar{B} + C)(A + \bar{C}) + \bar{A}B + C(\bar{A} + B)$$

3. Boolean Algebra

Complete the label of the rightmost column in the following truth table.

A	B	C	$A \square \bar{B}$	$\bar{A} \square C$	$(A \square \bar{B}) \square (\bar{A} \square C)$
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	0	1	1

$$(A \square \bar{B}) \square (\bar{A} \square C)$$

4. Bit String Flicking

Evaluate the following expression:

$$(\text{LCIRC-3 } (\text{RSHIFT-2 } (\text{LCIRC-1 } 01101))) \text{ AND } (\text{LSHIFT-1 } (\text{RCIRC-2 } (\text{NOT } 11100)))$$

5. Bit String Flicking

Find all values of x (5-bits long) that make the following equation true.

$$x \text{ OR } (\text{LSHIFT-2 } 11011) = (\text{RCIRC-2 } 11011)$$

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1. Each time through the loop, the values of four elements of the array are toggled. The following table shows the elements of x that change from 0 to 1, and from 1 to 0, each time through the inner loop:

r	c	0→1	1→0
2	2	(1,1) (2,1) (3,3) (2,3)	
2	3	(1,2) (2,2) (3,4) (2,4)	
2	4	(1,3) (3,5) (2,5)	(2,3)
3	2	(3,1) (4,3)	(2,1) (3,3)
3	3	(3,2) (4,4)	(2,2) (3,4)
3	4	(2,3) (3,3) (4,5)	(3,5)
4	2	(4,1) (5,3)	(3,1) (4,3)
4	3	(4,2) (5,4)	(3,2) (4,4)
4	4	(4,3) (5,5)	(3,3) (4,5)

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2. The expression simplifies as follows:

$$\begin{aligned}
 &= (\overline{A}B A + \overline{A}B \overline{C} + AC + C\overline{C}) + \overline{A}B + (C\overline{A} + CB) \\
 &= \overline{A}B + \overline{A}B \overline{C} + AC + \overline{A}B + C\overline{A} + CB \\
 &= \overline{A}B + \overline{A}B \overline{C} + \overline{A}B + C(A + \overline{A} + B) \\
 &= \overline{A}B(1 + \overline{C}) + \overline{A}B + C(A + \overline{A} + B) \\
 &= \overline{A}B + \overline{A}B + C \\
 &= A \oplus B + C
 \end{aligned}$$

$$A \oplus B + C$$

The following expressions are also correct: $C + A \oplus B$, $C + B \oplus A$, and $B \oplus A + C$.

No parentheses are required, since XOR has higher precedence than OR. The operands to the XOR and to the OR may be commuted.

3. In the fourth column, the expression $A \square \overline{B}$ is true only when $A = 1$ and $B = 0$, so the operator must be an AND. In the fifth column, the expression $\overline{A} \square C$ is true except when $A = 1$ and $C = 0$, so the operator must be an OR. The rightmost column is the OR of the previous two columns.

$$(A \square * \overline{B}) \square + (\overline{A} \square + C)$$

4. Simplify the left operand of the AND

$$(LCIRC-3 (RSHIFT-2 11010)) = (LCIRC-3 00110) = 10001$$

and now the right operand:

$$(LSHIFT-1 (RCIRC-2 00011)) = (LSHIFT-1 11000) = 10000$$

Finally, $10001 \text{ AND } 10000 = 10000$.

10000

5. Simplify the equation by evaluating the LSHIFT and RCIRC:

$$x \text{ OR } 01100 = 11110$$

Now, rewrite x as abcde and consider each bit in turn.

- a OR 0 = 1 \Rightarrow a = 1
- b OR 1 = 1 \Rightarrow b = anything
- c OR 1 = 1 \Rightarrow c = anything
- d OR 0 = 1 \Rightarrow d = 1
- e OR 0 = 0 \Rightarrow e = 0

10010, 10110, 11010, 11110

Can be written as $1**10$.