Recursive Functions

Find $f(12)$ given $f(x) = \begin{cases} 2 * f(x-3) - 3 & \text{if } x > 6 \\ f(x+2) + 1 & \text{if } 4 < x \le 6 \\ x + 4 & \text{if } x \le 4 \end{cases}$

Answer: 95

$f(12) = 2 * f(9) - 3$
$f(9) = 2 * f(6) - 3$
$f(6) = f(8) + 1$
$f(8) = 2 * f(5) - 3$
$f(5) = f(7) + 1$
$f(7) = 2 * f(4) - 3$
$f(4) = 4 + 4 = 8$

Now, substitute back:

$f(7) = 2 * f(4) - 3 = 2 * \boxed{8} - 3 = 13$
$f(5) = f(7) + 1 = \boxed{13} + 1 = 14$
$f(8) = 2 * f(5) - 3 = 2 * \boxed{14} - 3 = 25$
$f(6) = f(8) + 1 = \boxed{25} + 1 = 26$
$f(9) = 2 * f(6) - 3 = 2 * \boxed{26} - 3 = 49$
$f(12) = 2 * f(9) - 3 = 2 * \boxed{49} - 3 = 95$

Computer Number Systems

Which of the following 5 numbers is the largest? $F1_{16}$, 375_8, $10F_{16}$, 264_{10}, 11111000_2

Answer: $10F_{16}$

The straightforward approach to solving this problem is to convert each number into a common base, and compare those values. We'll convert each to base 10:

$F1_{16} = 241$

$375_8 = 253$

$10F_{16} = 271$

$264_{10} = 264$

$11111000_2 = 248$

Prefix/Infix/Postfix

Evaluate the following postfix expression if a = 3, b = -2, and c = 5

```
a b – c / b ^ a – c 2 a * 3 b * – * +
```

Answer: 58

```
a b – c / b ^ a – c 2 a * 3 b * – * +
     = (3 (-2) -) 5 / (-2) ^ 3–5 (2 3 *) (3 (-2) *) – * +
     = (5 5/) (-2) ^ 3 - 5 (6 (-6) –) * +
     = (1 (-2) ^) 3 - (5 12 *) +
     = (1 3 –) 60 +
     = -2 + 60
     = 58
```

LISP

Evaluate the following LISP expression:

```
(ADD (MULT 3 (EXP 2 3))(MULT(EXP 2 2)(SUB(MULT 5 3) 6))(EXP 2 (SUB 9 3)))
```

Answer: 124

Evaluate from the inside parentheses to the outside, simplifying one expression on each left-to-right pass through:

```
=> (ADD (MULT 3 8)(MULT 4 (SUB 15 6))(EXP 2 6))
=> (ADD 24 MULT 4 9) 64)
=> (ADD 24 36 64)
=> 124
```

Boolean Algebra

Simplify the following Boolean expression: $A(\overline{A}+B)+\overline{A}B$

Answer: B

Apply the distributive law to the left term: $A(\overline{A}+B)+\overline{A}B = A\overline{A}+AB+\overline{A}B$

And now simplify: $0+AB+\overline{A}B = B(A+\overline{A}) = B$

What Does This Program Do?

What numbers are printed after the following program is executed?

```
C = 0: S = 0
FOR I = 1 TO 50
     IF I/2 = INT(I/2) THEN C = C + 1 ELSE S = S + 1
NEXT I
FOR K = 1 TO 50
     IF (K/3 = INT(K/3)) AND (K/2 <> INT(K/2))
          THEN C = C + 1 ELSE S = S + 1
NEXT K
FOR J = 1 TO 50 STEP 2
     IF (J/5 = INT(J/5)) AND (J/3 <> INT(J/3))
          THEN C = C + 1 ELSE S =S + 1
NEXT J
PRINT C, S
END
```

Answer: 36 and 89

The first loop looks at numbers between 1 and 50. If the number is divisible by 2, then C is incremented; otherwise S is incremented. Thus, after the first loop, C has a value of 25, and S also has a value of 25.

The second loop, on K, also considers the numbers between 1 and 50. If the number is divisible by 3 and also not divisible by 2 (in other words, a multiple of 3 that is not a multiple of 2, namely 3, 9, 15, 21, 27, 33, 39, 45), then C is incremented. Otherwise S is incremented. Thus, after this loop, C is 25+8=33 and S is 25+42=67.

The third loop, on J, looks at the odd numbers between 1 and 50 (i.e., 1, 3, 5, …, 49). If the number is divisible by 5 by not by 3, C is incremented; otherwise, S is incremented. Thus, C is incremented when J is 5, 25, and 35; and S is incremented 22 times, when J is 1, 3, 7, 9, … 49.